Instructors: **1.** Dr. Rola Alseidi	**Philadelphia University** **Faculty of Science** **Department of Mathematics** **Midterm Exam**	**Academic Year:** 2022-2023 **Semester:** Fall **Date:** 11/12/2022 **Course:** Linear Algebra (2) **Duration:** 75 Min

Name: **I.D. Number:**

Question One: [6 points]

1. Consider the set

$$V = \{(x, y) : x, y \in \mathbb{R}\}$$

and define the following operations on V

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$k \odot (x_1, y_1) = (0, ky_1),\ k \in \mathbb{R}$$

Find

(a) $(-1, 2) \oplus (0, 1)$.

(b) $-2 \odot (1, -1)$

(c) Is there a zero vector in this space. Explain

(d) Is V a vector space. Explain

2. Let W be the set of all 2×2 matrices of real entries of the form

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

such that $b \geq 0$.
Is W is a subspace of the vector space $M_{2\times2}(\mathbb{R})$ with standard matrix addition and scalar multiplication.

Question Two: [6 points]

If the matrix

$$R = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

is the reduced row echelon form from the matrix

$$A = \begin{bmatrix} 0 & 2 & 2 & 4 \\ 1 & 0 & -1 & -3 \\ 2 & 3 & 1 & 1 \\ -2 & 1 & 3 & -2 \end{bmatrix}.$$

Find

1. A basis for the row space of A.

2. A basis for the column space of A.

3. A basis for the null space of A.

4. The rank of the matrix A.

5. The nullity of the matrix A.

Question Three: [5 points]

Show that the set $S = \{1 - 2x,\, x - x^2,\, -2 + 3x + x^2\}$ is a linearly independent set.

Question Four: [5 points]

Find the coordinate vector of $v = (2, -1, 3)$ relative to the basis $S = \{(1, 0, 0), (2, 2, 0),\, (3, 3, 3)\}$.

Question Five: [2 points]

If A is a 3×5 matrix with rank $= 2$. Find

1. Rank A^T.

2. Nullity of A^T.

Question Six: [6 points]

Circle True or False. Read each statement carefully before answering.

(a) True False The set $\{\sin^2(x), \cos^2(x), \cos(2x)\}$ is linearly independent.

(b) True False The null space of 3×4 matrix always has dimension at least one.

(c) True False If V is a 5-dimensional vector space, then every collection of five vectors in V is linearly independent.

(d) True False The Set $S = \{(1, 2, 5), (1, 3, 7), (1, -1, -1)\}$ is a basis for $\mathbb{R}^3$

(e) True False Let A be 3×3 matrix with rank 2, then the homogeneous system $Ax = 0$ has infinity many solutions.

(f) True False Any subset of a vector space must be a vector space .

Good Luck